So, I have a method to solve systems of equations that's a lot like what the algebra kids (and the rest of the world) call substitution, but it's a little different. It's not as fast or cool, but it is a good option for kids who have algebra skills that aren't perfect yet. Here are a few examples to show how it works.

EXAMPLE 1: Solve the system of equations.

$$\mathbf{y = 3x + 1 \qquad y = 2x + 2}$$

The whole goal of solving a system of equations is to find an x-value and a y-value that work in both of the given equations. In this one, the solution is $x = 1$, $y = 4$ … because those values of x and y make both equations true. (Don't tell anyone that you already know the answer).

Here is the logic that we'll use in this method. In the first equation, we can see that y is equal to $3x + 1$. In the second equation, we can see that y is equal to $2x + 2$. So, if y is equal to $3x + 1$ … and it's also equal to $2x + 2$, it must be true that $3x + 1$ must be equal to $2x + 2$ as well. So, we write

$$3x + 1 = 2x + 2$$

As you write that, we should be at a point where things look familiar. You can solve this equation the same way you've been doing other equations in class.

$$3x + 1 = 2x + 2$$
$$x + 1 = 2 \qquad \textit{throwing the 2x to the left side}$$
$$x = 1 \qquad \textit{throwing the 1 to the right side}$$

Unlike the equations you've been doing in class, though, you're not done. You need to know the value of x **and** y … not just x. But we can use our value of x to find the value of y. Just put the value of x into one of the original equations; pick whichever one you think looks easier. If, for example, I use the top one, substituting my x-value looks like this:

$$y = 3(1) + 1 = 3 + 1 = 4$$

So, $x = 1$ and $y = 4$. We most often write that as a point: **(1, 4)**.

EXAMPLE 2: Solve the system of equations.

$$\mathbf{2x + y = -1 \qquad x + 3y = 12}$$

If you'd like this one to be as easy as the last one, you'll need to make the equations look like they did in the last one. That means we'd like to rewrite them both so that y is alone on one side of each equation. On the first equation, we just need to throw the 2x to the right side. We'll do the same trick with the second equation, throwing the x to the right side. Remember that both x terms will become their opposites as they go.

$$y = -2x - 1 \qquad 3y = -x + 12$$

But things still aren't quite how they were in the first example. At this point, we'd like to say that the left sides match … but they don't. In the first one, the left side is y … in the second one, the left side is 3y. Our logic trick requires that they match.

To fix that, we'll multiply every term in the first equation by 3.

$3y = -6x – 3 \qquad 3y = -x + 12$

Now they match! We can say that if 3y equals -6x – 3 and 3y also equals –x + 12, those two things must equal each other as well. So, we can write

$-6x – 3 = -x + 12$	
$-3 = 5x + 12$	*throwing the -6x to the right side*
$-15 = 5x$	*throwing the 12 to the left side*
$x = -3$	*dividing by 5*

And like before, we're not done until we find both x and y. We'll put our x-value into one of the original equations – let's use the second one this time.

$(-3) + 3y = 12$	
$3y = 15$	*throwing the -3 to the right side*
$y = 5$	*dividing by 3*

Therefore, our solution is **(-3, 5)**.

EXAMPLE 3: Solve the system of equations.

$\mathbf{3x + 5y = 19} \qquad \mathbf{4x - 7y = -43}$

On the first two examples, we made the equations match with y … but you're allowed to match them at x if that's easier. But this set doesn't look like we're close to a match at all. Let's make them match with x. To begin that, I'll clear the left sides out on both equations for x. Throwing the 5y and the -7y gives

$3x = -5y + 19 \qquad 4x = 7y – 43$

The best way to make them match might be to get both of them to 12x; that means multiplying every term in the first equation by 4 and every term in the second equation by 3.

$12x = -20y + 76 \qquad 12x = 21y – 129$

Our logic trick lets us combine those into something we can solve.

$-20y + 76 = 21y – 129$	
$76 = 41y – 129$	*throwing the -20y to the right side*
$205 = 41y$	*throwing the -129 to the left side*
$y = 5$	*dividing by 41*

And substituting that into, say, the first equation gives

$3x + 5(5) = 19$	
$3x + 25 = 19$	*simplifying the multiplication*
$3x = -6$	*throwing the 25 to the right side*
$x = -2$	*dividing by 3*

Therefore, our solution is **(-2, 5)**.