You Should Really Get to Know Mathew

A Seedbed for Incorporating
Mathematics into Every Classroom

You Should Really Get to Know Mathew

A Seedbed for Incorporating Mathematics into Every Classroom

The End

Chapter 1

The end has been widely agreed upon for some time. Whether it was born from a sweeping goal of providing a more complete educational experience or a narrower focus on improving standardized test scores, schools have been charged with the task of offering an interconnected education. Especially as it has pertained to English and mathematics instruction, schools are looking to reduce teaching in isolation and weave curriculum together across all disciplines.

The means, however, have not been as broadly accepted. Some schools have organized ongoing school-wide writing projects, often to be completed and assessed outside of English classes. Others have seized students' existing homeroom time to meet the sixty minutes of daily instruction advocated by the National Council of Teachers of Mathematics. Still others have shifted their efforts to making small or large changes in science, art, general music, and other classes to bolster English and math instruction.

With a full curriculum and accountability looming, how can a social studies teacher be expected to teach English and math as well? The question is asked considerably more often than it is answered. Jealous math teachers will confirm that English has had a far greater path of acceptance; teachers of all disciplines have found ways to incorporate reading assignments, written projects, and class presentations that support English standards.

But incorporating mathematics? Outside of science and industrial technology, teachers in many disciplines have found meaningful connections hard to come by. In many ways, the whole process is not unlike a man introducing his sister to a coworker in whom he thinks she may have interest. Although honesty is best, a hesitant audience sometimes necessitates a little creativity.

The following pages feature five approaches for including mathematics across the curriculum, framed in a matchmaking scenario between a well-meaning brother, a skeptical sister, and coworker Mathew. Whether selling Mathew or mathematics, some approaches are more successful than others. Although there's no blanket plan for successfully merging existing standards with mathematical content, the following ideas may provide teachers with some ideas for small changes that may pay large dividends for students.

Isolation

Chapter 2

What the Brother Might Choose to Say

I know that I invited you over to watch the Super Bowl, but I'd like to turn the television off and spend our four hours together talking about Mathew instead. There will be another Super Bowl next year.

What the Sister Might Choose to Think

We both enjoy the Super Bowl … why would we trade it in for this? And what makes this such a big emergency all of the sudden? Is having this discussion something that someone is making you do? I guess I'll have my answer if we talk about Mathew today and then never again. *You know, I think we'd both be much better off spending our time on what we're scheduled to do together.*

As ridiculous as that approach seems, it's remarkably similar to the plan to integrate math in some schools. A few weeks before the standardized testing period begins, some schools choose to initiate a plan like one of the following:

- Every class in the building starts with two fraction problems.
- A half-page math skills review becomes homework in music classes.
- All teachers on staff conduct a mini-lesson on mean, median, and mode with their students.

Although some math skills are strengthened by these activities, students may learn other unintended lessons as well:

Math is more important than my other subjects. That false message is sent when, at the important time of year, we tell students that we don't have as much time for other subjects … math needs us.

Doing math is a punishment for us. When kids hear their parents say "You can only play with the neighbor boy for thirty minutes today," it's apparent that playing with the neighbor boy is a fun experience. However, "You have to play with the neighbor boy for at least ten minutes today" sends an entirely different message.

My teacher doesn't really want to include math in our class. It's unlikely that every teacher will seize the chance to add these canned activities in class and embrace it in such a way that students will see it as a pleasure. Like taking out the trash at home, teaching math might instead appear to be a job so heinous that it has to be spread around the staff. But no matter how well the teacher handles the temporary policy, what other conclusion can students reach when the extra math exercise is dropped as soon as the standardized testing period ends?

Math has nothing to do with anything else I do all day. The above plans of *including* math content in other classes don't include math in instruction at all; at best, the plans divide each class meeting into two classes: a math review class and a class that represents the real reason you're together.

If the goal is to provide an interconnected education, the content must be connected. The good news is that mathematics would no longer be around as an intellectual pursuit if it didn't apply to so many things around us. The goal, then, is to integrate mathematics into what is already being done. Here's a quick test for teachers to determine what level of integration of math content is present:

Today, we are doing math as part of my class **instead of doing** what I need to do with students.

Today, we are doing math as part of my class **to supplement** what I need to do with students.

Maybe the brother and sister will have the idea to invite Mathew over to watch the Super Bowl with them. If so, would it make sense to have a portion of the game where Mathew watches alone while the siblings leave the room and vice-versa? Instead, the hope is that if all three are in the room together, Mathew is included in the existing activity … he adds to it rather than serves as a substitute for it.

While every math teacher would love it if students had seven math classes per day, even they realize that students and teachers will *both be much better off spending their time on what they're scheduled to do together.*

Immersion

Chapter 3

What the Brother Might Choose to Say

I know that I told you I'd come over on Saturday to test out that tandem bike you found online … I was thinking of inviting Mathew to come out with us! What would you think about that?

What the Sister Might Choose to Think

We need a third person to ride a tandem bike? *Mathew may be great, but that doesn't mean we have to include him in everything we do together.*

A few years ago, a grade-level team was presented with a canned interdisciplinary unit focused on the Civil War. Obviously, the social studies connections were abundant. A study of a period-appropriate novel was included for use in English class; the unit also contained some material on Civil War medicine for science classes to discuss.

The math component, however, was a single worksheet on which students were given necessary data and asked to calculate what percent of the soldiers killed in each battle were from the North and what percent were from the South. What was intended to be a full-day's plan for a class of eighth graders might have kept a sixth grader and his calculator busy for five minutes.

A good math teacher can likely make a decent connection between mathematics and many lessons in many other disciplines. When a connection is not apparent or convenient, however, teachers need to give themselves permission to leave it out.

When we instead force lazy connections between mathematics and other disciplines, we teach students a different lesson: most of what you'll need mathematics for in the real world could be accomplished by a calculator and what you learned through elementary school. That attitude cheapens the study of mathematics in general and weakens the secondary curriculum specifically.

Students are learning new mathematical concepts and procedures every day; those are the ones we need to reinforce when opportunities present themselves.

We'll take every chance possible to teach students the natural beauty of principles of mathematics showing up in even the strangest of places. But if we instead warp this wide-reaching utility into "every situation can be turned into an addition problem," we've traded the message of beauty for one of mundane omnipresence or inescapability … like sales tax, laundry piles, television commercials, and anything else that we see every day but aren't particularly happy about.

Teachers in other disciplines should check current math standards or have a discussion with a math teacher about what connections could allow students to practice their emerging skills in a new setting. If connections are found, great! If not, they shouldn't be forced. After all, *math may be great, but that doesn't mean we have to include it in everything we do together.*

Value

Chapter 4

What the Brother Might Choose to Say

Do you remember that concert that I couldn't get us tickets for? I just mentioned something in passing about it to Mathew, my friend at work, and he made a few calls. It turns out that he was able to cash in a favor from a guy with two extras … Mathew said that we're welcome to them!

What the Sister Might Choose to Think

Mathew seems very generous and considerate … and a lot better at getting things done than my brother, that's for sure. *Maybe this isn't a life-changing event, but it seems true that knowing Mathew can make good things happen.*

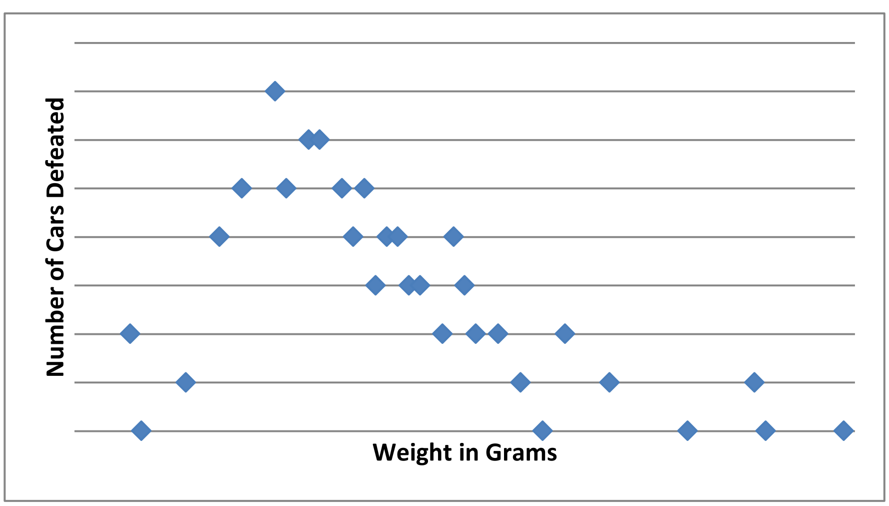
Granted, the ending seems a bit optimistic … but the romanticized point is clear. People are interested in things that hold value to them.

Many middle school industrial technology students do a project involving shaping a CO_2–cartridge car out of a block of wood. After cars have been constructed, teachers often organize a race in the style of a class tournament; students with faster cars make their way through the bracket until a class champion is declared.

As students begin the project, teachers generally reiterate design principles that have their basis in material the class has already discussed: the lighter your car is, the faster it will be … but, if you choose to make it too light, you run the risk of it breaking under the force of ignition.

It may be that the previous statement is the only advice students receive; from that point, it is left to their own judgment to strike a balance between speed and strength. Consider, however, what would happen if the teacher provided students with a bit more information, perhaps phrased like this:

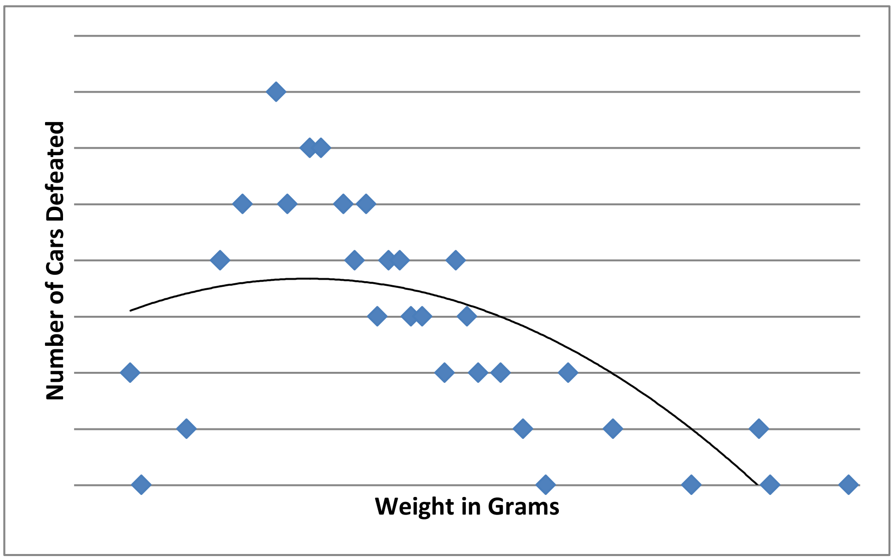
As you design your cars, I thought you might be interested in seeing this:



This is a graph of the results of the car races from the last couple of semesters. I used Excel to graph the weight of each student's car on the horizontal axis and the number of cars each car defeated on the vertical axis. In other words, if a sixty-gram car beat three cars in the tournament, that data point will show up on the graph at (60, 3).

Like we talked about earlier, you can see that super-light cars often snap after a few races; super-heavy cars stay strong, but are too heavy to win a lot of races.

Excel also has a feature that will mathematically describe your data as best it can with a curve. When I did that, Excel showed me this:



According to Excel, this best-fit curve is given by the equation $n = -0.002w^2 + 0.232w - 2.211$, where *n* stands for the number of cars defeated and *w* stands for the weight of the car in grams. So, if you'd consider it to be worth your time to do so, you could plug some possible values in for *w* to see what kind of a result you might expect for *n*.

You may have noticed that the axes on both graphs were titled but not marked with specific *w* and *n* values; this would be intentional on the part of the teacher. Marking the graphs with specific values changes students' job from exploring the equation to simply reading a data value off of the graph. In other words, instead of *doing* an investigation in mathematics, students would simply be lifting the results of the *teacher's* investigation in mathematics.

As left by the teacher at this point, the project now contains a mathematical component. An opportunity to use real data in an authentic way has been supplied, along with the option for students to disregard the data and equation, if they so choose.

Teachers who wanted to add an extra layer to the process could plan something like this:

When a best-fit curve like this one is calculated with Excel or any other method, there is a certain degree of symmetry present in the graph. Because of what I know about the car race, however, I wasn't satisfied with that.

Do you see how the values on left side of the graph rise a lot more quickly than the values on the right side of the graph fall? It's kind of like a slide at the park; you go up a steep ladder, but then come down a gentler slide. From what I've seen in past races, I know that if you make your car too light, there's no gentle slide … those cars start snapping more and more.

To try to account for this asymmetry, I went back to Excel and broke my data into three sets based on the weight of the car. Instead of one simple equation, I now have something called a piecewise function:

$$n = \begin{cases} 0.011w^2 - 0.554w + \quad 6.242 & if\ w < 50 \\ 0.005w^2 - 0.880w + 36.871 & if\ 50 \leq w \leq 75 \\ 0.001w^2 - 0.214w + 11.388 & if\ w > 75 \end{cases}$$

This thing is much less complicated than it looks; it just says for you to use the top expression if you're testing a weight value below 50 grams, the middle expression if you're testing a weight value between 50 and 75 grams (inclusive), and the bottom expression if you're testing a weight above 75 grams.

If this piecewise function scares you, go back to the symmetric equation I showed you earlier. If that scares you as well, just use your best judgment on where to put the weight of your car. The decision is up to you … but I'll bet you can guess which method I'd use.

And at that point, the design, production, and testing components of the project resume as usual. As the tournament ends, some informal discussion could bring forth some closure and hammer these auxiliary lessons home:

Data equations help make good predictions, but they are still only predictions. It's not likely that the results of the current race perfectly mirror the previous data and resulting equations. Why is that? While weight may be the most important predictor of a car's performance, it isn't the only predictor; how the weight is distributed and balanced is an important factor as well.

People are sometimes smarter than technology. Through the development of the piecewise function, students have seen an example of a teacher judging a computer's initial work as unsatisfactory in context.

Collecting more data helps achieve better results. Teachers who end the unit by collecting current data to be assimilated into the existing data bank will be making that clear to students.

Students who spend more time planning have more success. Hopefully, a quick survey following the tournament will verify that those students who spent some time with the equations generally fared better in the tournament than students who chose to go by feel instead.

Through the project, the teacher and students shared the mathematical load. In this specific example, it's true that the teacher did most of the heavy lifting. But even if students never actually saw Excel at work or have no idea where the associated equations came from, many have had an opportunity to learn firsthand that *it seems true that knowing math can make good things happen.*

Truth

Chapter 5

What the Brother Might Choose to Say

Have I ever told you about Mathew, sis? He is the funniest guy I've ever met … and the smartest … and the most generous … and the best dancer … and the most steadfast recycler … and the kindest to old people …

What the Sister Might Choose to Think

The description of this guy is obviously too good to be true. As soon as I get proof that it's just a list of hyperbole, I won't trust my brother about Mathew (or anything else) again. You know, why wait for proof? I think I'll just scratch Mathew off the list. *Your dishonesty about Mathew results in my distrust of Mathew.*

At this point, a couple of words of truth need to be spoken.

Applying math concepts in new situations isn't always easy for kids. Although the experience varies from school to school, students have received a large amount of their previous math instruction and practice through very direct methods. A math teacher discusses a concept, students practice thirty problems that directly and overtly apply to that concept, and everybody shows up the next day to do it all again.

Alas, math in the real world doesn't follow that model. Kids aren't headed for jobs where the boss walks in and demands, "Thompson, get me the odd problems from 1-45 on my desk by tomorrow morning."

Instead, the math that people do in their jobs or their personal lives is sometimes hidden inside a story about a problem someone is having or a need to determine the best course of action in a certain situation. That story must be decoded, with relevant details plucked from irrelevant information and a list of constraints determined and followed. An action plan is chosen, and then sometimes defended to coworkers or family members. After a solution is found and checked, it is most often explained to a boss or client before it can be used.

Like enrolling a sprinter in a marathon, a change of pace is necessary. It's not that the sprinter isn't capable of running the longer race … he may find that he runs it very well. But the fact that the race does not perfectly match the training is a fact that can't be overlooked. And if the change is not communicated properly by the coach, he'll lose his sprinter before the first turn. Unfortunately, no matter the preparation, some still really like to see that finish line from the starting block. For that reason, …

Applying math concepts in new situations isn't always easy for teachers. In the previous chapter, would it be easier for the industrial technology teacher to simply direct students to a target weight of forty-nine grams for their cars? In a big way, the answer is yes. The teacher would avoid the tasks of collecting past data, fitting data into useable equations, explaining the process to students, coming alongside students who are having trouble, saving the materials for next semester's class, and developing the entire idea in the first place. In exchange for **not** having to do those tasks, the teacher is **also** guaranteed that every student knows the correct target weight.

It's also easier to do the Wednesday crossword after the Thursday paper comes out.

The point here is that simple *completion* is not the exercise; *competence* is the exercise. Whether it's expressed as "no pain, no gain," "teach a man to fish," or some other aphorism, the job of guiding students through uncomfortable situations as learning experiences is what all teachers do every day. It's much easier to recount the events from a novel than to discover the symbolism behind them; it's faster to read about a catalyst's role in a chemical reaction than to demonstrate it; it takes a lot less effort to tell *when* a war started than explain *why* it started. And even if a school has a math program that already cultivates this roll-up-our-sleeves attitude toward mathematics in its students, authentic tasks written at appropriate levels will always take time and effort on behalf of both students and teachers. And that's OK.

If this booklet had been written from the perspective that introducing mathematics into other subjects is always easy and fun, it would be immediately rejected as out of touch with its intended audience. Likewise, teachers who are seeking to encourage students to work in different (and sometimes demanding) ways need to be honest with them as well. After all, *dishonesty about math results in distrust of math.*

Ubiquity

Chapter 6

What the Brother Might Choose to Say

Hey, sis … did I tell you about the new couch I got? Last Saturday, Dad and I picked up Mathew and we went to the flea market downtown. We looked around for hours before Dad spotted a couch that looked just like the one we had in the family room when we were kids. We took Mathew back home to get his truck, and the couch just barely fit. You should come over and see it sometime!

What the Sister Might Choose to Think

Who is this Mathew person? Why does my brother refer to Mathew as if I already know him? Should I know him? Apparently Dad knows him. Maybe I should know Mathew. *Whether I've realized it or not, Mathew is already a part of things around me.*

One way of teaching students to solve word problems is to have them do a direct translation of the individual parts of sentences to components of equations. For example,

What is 40% of 80?

becomes

$$w = 0.4 \times 80$$

Math students are generally more comfortable with relationships presented in the form of equations. When the problem is just a bit more complicated,

Bob weighs 100 pounds. His weight is ten pounds more than twice Fred's weight. How much does Fred weigh?

the translation is not as apparent. A teacher may point students to the second sentence of the problem … and then ask what the antecedent for "His" is. When the antecedent is identified to be Bob, the translation becomes

$$100 = 2F + 10$$

Asking a math class about the antecedent for a pronoun in this instance accomplishes three things. First and foremost, it helps the math teacher and the math student reach their shared goal of solving the problem.

Second, it helps reinforce a concept from English class. In no way is the math teacher *teaching* what an antecedent is; students have already learned the term from an expert. Instead this carefully-selected word turns a moment that could easily have been lost into a microscopic English review.

Third, it lends support to the English teacher's cause in general. We know that there are students in every class that view the subject matter as something that "she's making us learn." In other words, knowing the vocabulary, skills, and concepts are important while in that room … but that's about it. Hearing a math teacher talk about an antecedent puts some doubt into that conclusion. What's more, the math teacher didn't talk about antecedents because standardized tests are next week or because today is "English Day" in all the classes; he just mentioned the term because educated people use it.

Using the term is intentional by the teacher, but seems coincidental to the students. It's not a sales pitch that students accept or reject; acceptance is assumed. The power is in the simplicity.

This was an example of integrating a term from English class into a math class; is it possible to draw a parallel in the other direction? You probably already detected it, but the previous question just did exactly that.

As a matter of fact, there have been several similar instances throughout this text … instances where a word that math teachers would consider to be their own has been used in a more general context. If you noticed them, good for you. Of course, if you didn't notice them, that's even better … it's an indication that you've realized that *math is already a part of things around you.*

Beginnings: Speaking the Language

Chapter 7

As demonstrated in the previous chapter, incorporating mathematics content into non-math classes through vocabulary is a terrific place to start for many reasons: authentic opportunities are plentiful, no substantial changes to existing lessons are required, and lengthy collaboration with a math teacher isn't necessary. Listed below are some examples of general math terms that would have some crossover value to other disciplines.

absolute value
adjacent
altitude
angle of elevation
axes
axis of symmetry
bisect
box-and-whisker plot
central angle
chord
circumscribed / inscribed
coefficient
collinear
commutative
compound inequality
concave / convex
congruent
conjecture
continuous / discrete
converse
correlation
corresponding
counterexample
dependent / independent variable
dilation
direct / inverse variation
domain / range
empty set
equiangular
equidistant
equilateral
function
horizontal / vertical shift
hypotenuse
if-then statement
image / pre-image
intercept
interquartile range
intersection / union
interval
inverse
linear
midpoint
mutually exclusive
origin
outlier
prism
proportional
quadrant
quartiles
rate of change
reflection / rotation
regular polygon
sample space
scale factor
scatterplot
scientific notation
sector
similar
slant height
slope
stem-and-leaf plot
substitution
system of equations
tessellate
translation
truncate
Venn diagram
vertex
weighted average

Using correct terminology in mathematics is a cornerstone of developing a sense of precision in students. As in any area of study, there exist a few families of related terms and concepts that mathematics students sometimes confuse and mistakenly interchange; a few common missteps that might pop up outside of math class are listed below.

Circle vs. Sphere / Square vs. Cube

Some students tend to refer to three-dimensional objects by the shapes of their shadows. Of course, circles and squares are two-dimensional; spheres and cubes are three-dimensional.

Divided By vs. Divided Into

Six divided by three is two; six divided into three is one-half.

Equation vs. Expression …

The sentence $3x + 2 = 5x – 6$ is an equation: a mathematical sentence that says two expressions are equal. An equation is a complete sentence; equals is the verb. The $3x + 2$ and $5x – 6$ components are expressions: mathematical phrases that describe quantities. An expression is a sentence fragment.

… and Solve vs. Evaluate

To solve means to find the solution; to evaluate means to give a numeric value of an expression. In short, equations are solved; expressions are evaluated.

Factor vs. Multiple

The factors of 12 are 1, 2, 3, 4, 6, and 12. The multiples of 12 are 12, 24, 36, 48, and so on.

0.99¢ vs. $0.99

Students sometimes write the first (a bit less than a penny) when they mean the second (a bit less than a dollar).

Probability vs. Odds

These terms are not interchangeable. Probability is a ratio of the total number of ways to succeed compared to the total number of possible outcomes; the probability of rolling a two on a die is one-sixth. Odds is a ratio of the total number of ways to succeed compared to the total number of ways to fail; the odds of rolling a two on a die are one to five.

Quadrilateral vs. Rectangle vs. Square

A quadrilateral is a four-sided polygon; a rectangle is a quadrilateral with four right angles; a square is a rectangle with four equal sides. Students sometimes classify these objects as three separate items instead of the Russian nesting dolls that they are.

This list is in no way exhaustive; teachers also often see students make smaller confusions like using the term *line* in place of *line segment*, a plus sign when an ampersand is appropriate, or the improper use of an equal sign as a mere “step separator” as demonstrated below.

Bob has three red marbles and two blue marbles. Fred gives Bob a marble to match each marble in his original collection and four new yellow marbles. How many marbles does Bob have now?

$$3 + 2 = 5 \times 2 = 10 + 4 = 14$$

Although the final number of marbles is correct, the author of this solution has used the equal sign to declare some unequal things equal.

The above list also speaks nothing of students who use the mathematically-imprecise term of *diamond* to describe something that someone standing in a different location would properly classify as a rhombus or students who have defined something that does not fit their standard for beauty or stability as an *upside-down triangle*. And unless you want a ten-minute lecture from a mathematician, don't follow society's recent convention of hijacking incredibly precise mathematical terms such as *exponential growth* or *random* to take the place of general ideas like "grew faster than I would have expected" or "unusual in my opinion," respectively.

Many of these clarifications seem picky to students … and perhaps to other teachers. But the process is no different from the perpetual battles that teachers of other disciplines fight: British vs. English, concrete vs. cement, weight vs. mass, less vs. fewer, sketch vs. draw, and so on. In any subject, at the core of the pickiness is precision. Using correct terminology helps students clarify their thinking, make distinctions between similar concepts, and communicate effectively. The more often students see precision modeled throughout the school day, the more they will assimilate it into their own processes.

Beginnings: Cultivating Number Sense

Chapter 8

Good writers are often born from good readers. Students who read a lot tend to develop an innate ability to put their own thoughts together comfortably. It's not that they don't follow "the rules" of writing; it's that they have internalized language conventions so firmly that they follow them without a great deal of thought about the process.

The pursuit of cultivating number sense in students follows that same model. As students acquire more and more experience in mathematics, the hope is that they develop a confidence and automaticity that goes beyond merely following set algorithms.

To appreciate the width of the number sense continuum, examine how five different shoppers might react to a sign that advertises a $600 television being sold at 30% off:

Shopper A: Let me get my phone. OK, 600 × 30 = 18000. That's not right. Maybe it's 600 ÷ 30. That's 20. I would have guessed that the discount was more than $20 off, but that's what the calculator says, so it must be right.

Shopper B: Let me get my phone. Since 600 × 0.3 = 180, the discount must be $180. To subtract 180 from 600, I could subtract 200 and then add back 20. So, the TV sells for $420 now.

Shopper C: Thirty percent off is just about a third off. A third of the original $600 is $200; that would leave the sale price to be $400. Since the discount is a little bit less than a third, I'd be paying a little more than $400 for the TV.

Shopper D: I'm going to ignore the zeroes and decimals for a minute. Since 6 × 3 is 18, the discount must be $18 or $180 or $1800 or $1.80 or whatever makes sense. With a 30% discount on a $600 item, the only thing that makes sense is $180 for the discount. The sale price must be $420.

Shopper E: If they're giving me 30% off, I'm still paying the other 70%. Since 7 × 6 is 42, we must be talking about $420.

Five shoppers traveled five different paths, and four of them reached a useable answer in a reasonable way. Which successful method is best to teach to students? The answer is ***all of them***. With a diverse mathematical tool belt, students have the luxury of using whatever method makes sense to them and seems appropriate for each new situation that presents itself.

Every class in a school likely has a moment when there's a reason to add three-fourths of something to one-half of something … or to know how many right answers on an eight-question quiz would constitute a passing grade … or to have an idea of how long or heavy something is without measuring it … or to provide an estimated ratio between two large or ugly numbers. Students who witness these numbers wrangled without a calculator, either by a teacher or a fellow student, have seen number sense in action. Better yet, they've seen number sense in action ***outside the math classroom***. The hope is that these students not only adopt the specific procedure put in front of them, but also increase their desires to develop their own skills.

Beginnings: Fostering the Culture

Chapter 9

Many years ago, curriculum standards in mathematics were almost entirely skill-based. Higher-order skills of proof, communication, and problem solving were eventually added, but as an independent unit of study. Now, teachers have come to agree that these higher-order skills should not be introduced as independent skills, but instead as a culture to be woven throughout the curriculum.

When the Common Core Standards were released in 2010, they contained a description of this intended culture, known as the *Standards for Mathematical Practice*. The eight components are listed below:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

A detailed description of these standards is available at corestandards.org. Guided by these standards, math teachers will continue to refine their methods in hopes of showing students daily examples of what it means to work like a mathematician.

To a large degree, this culture can carry over to other subjects as well. Teachers who incorporate mathematical vocabulary in their lessons are attending to precision; teachers who provide opportunities for students to use their number sense are allowing them to reason abstractly and quantitatively. And many other standards on the list might be reinforced with a single well-timed question in any class.

By way of example, suppose a teacher is having a discussion with a colleague, who says he's just finished reading a booklet about injecting mathematics into all classes … and considers it to be the most boring thing he's ever read. The teacher might ask, "What do you mean?" or, "Can you give me an example?" However, those questions technically don't speak to the nature of the claim. The colleague's assertion is not simply that the booklet *is* boring, it is that the booklet *bores at a rate that is unmatched by any other material*.

As a result, a better question might instead be, "Can you quantify that?" (This is the same concept that nurses in an emergency room use when they ask patients to rate their pain.) Putting some numbers to the claim will help substantiate it. The colleague might say

- "In nine chapters, I only saw two useable ideas."
- "It generally took about three sentences to convey what could have been said in one."
- "I fell asleep six times in reading it."

Although they don't appear in the traditional mathematical format, each answer above is, indeed, a rate: two-ninths of a useable idea per chapter, three written sentences per necessary sentence, and six sleeps per booklet. Therefore, a claim about a rate of boredom has now been substantiated by an actual rate of boredom.

Countless other questions like this one can be used to support this culture of mathematics. A dozen such prompts with example usage are listed below.

How can you put into perspective …

… the number of troops who died in that battle?
… what kind of pace the winner of the marathon kept?
… the number of kids in the United States who don't eat a proper diet?

What level of precision will we need when we determine …

… what volume of solution we need for this experiment?
… where to drill the hole?
… which European country has the highest per-capita income?

Beyond just giving an example, can you prove …

… which variable has the greatest impact on the experiment's result?
… that stretching out before exercising reduces the likelihood of injury?
… that this musician had an important impact on his field?

How could we use technology to help …

… analyze the choices you've made in your budget?
… determine which States will be the most important to watch on election night?
… display the data we obtained from this survey?

Is there a limit to …

… how high up we can scale the recipe given the ingredients we have left?
… how much we can dilute this paint?
… how heavy a satellite could be?

What are the constraints involved when …

… we choose a piece to perform at contest?
… we design a robot?
… we pick topics for the poems we write?

What pattern do you see in …

… where pioneers decided to settle?
… the structure of this piece?
… the steps of this dance?

Do you have enough …

… thread left to complete the bag?
… time to finish your novel if you keep reading at your current pace?
… data to make a valid conclusion?

What's the best way …

… to get everybody on stage for the concert?
… to load a projects into a kiln with limited space?
… to minimize waste when you cut pieces from this board?

Can you find a flaw in the logic in …

… this claim about cigarette smoking?
… the main character's plan?
… your classmate's response?

Do you see a relationship regarding …

… the amount of acid used and the speed of the reaction?
… the placement of the wings and the length of time the plane is in the air?
… how you hold or sharpen your pencil and the value of color you see?

What can you say about the converse of …

… "If a poem is a haiku, then it has three lines."
… "Artists create identities for cultures and societies."
… "If you do your homework every day, you'll pass this class."

Through thinking like a mathematician, students define structure, verify assertions, attend to precision, evaluate possible strategies, and exploit relationships. When teachers foster this culture outside the math classroom, it certainly needn't bring the class to a screeching halt; instead, the point of view can add an important layer to the existing curriculum, introducing a different and worthwhile perspective that may not have been present otherwise. Students benefit on both ends.

Q. E. D.

Some might consider it to be strange and backward to begin a booklet with an end and end it with beginnings. Others might consider it strange and backward to spend years training and planning to teach kids about the subject they love and then be asked to inject mathematics into it.

What may be even more strange and backward, however, is the thought that all of this is an effort to have students and teachers do additional work on behalf of mathematics … when, in actuality, mathematics is more than happy to pitch in on whatever work already exists.

Mathematics has not survived as an intellectual pursuit since early history because of what it does in isolation; its value lies in its universality of application and utility. No matter the task, a working knowledge of mathematics helps get things done.

Regardless of what some people say about him, that Mathew is a pretty good guy.